NATIONAL COMMITTEE FOR MATHEMATICAL CONTESTS

British Mathematical Olympiad

13th March, 1980

Time allowed - 3½ hours

Write on one side of the paper only. Start each question on a fresh sheet of paper. Arrange your answers in order.

Put your full name, age (in years and months) and school on the top sheet of your answers. On each other sheet put your name and initials.

Q 1. Prove that the equation $x^n + y^n = z^n$, where n is an integer > 1, has no solution in integers x, y, z, with $0 < x \leqslant n$, $0 < y \leqslant n$.

Q 2. Find a set $S = \{n\}$ of 7 consecutive positive integers for which a polynomial P(x) of the 5th degree exists with the following properties:

(a) all the coefficients in P(x) are integers;

(b) $P(n) = n$ for 5 members of S, including the least and greatest;

(c) $P(n) = 0$ for one member of S.

Q3. On the diameter AB bounding a semi-circular region there are two points P and Q, and on the semi-circular arc there are two points R and S such that PQRS is a square. C is a point on the semi-circular arc such that the areas of the triangle ABC and the square PQRS are equal.

Prove that a straight line passing through one of the points R and S and through one of the points A and B cuts a side of the square at the in-centre of the triangle.

Q 4. Find the set of real numbers a_o for which the infinite sequence $\{a_n\}$ of real numbers defined by

$$a_{n+1} = 2^n - 3a_n \qquad (n \geqslant 0)$$

is strictly increasing, i.e.

$$a_n < a_{n+1} \qquad (n \geqslant 0)$$

Q 5. In a party of ten persons, among any three persons there are at least two who do not know each other. Prove that at the party there are four persons none of whom knows another of the four.